MMSE Beamforming with Quadratic Quiescent Pattern Constraints for Circular Array STAP

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Abstract

The goal of this project was to develop new and innovative processing methods for clutter and interference mitigation for Space-Time Adaptive Processing (STAP) with circular arrays. The focus was on developing a robust minimum mean square error (MMSE) beamforming technique using quadratic quiescent pattern constraints (MMSE-QPC) that works with arbitrary array configurations, including the circular UHF Electronically Scanned Array (UESA) and the standard linear array. The MMSE-QPC technique provides a general framework for quiescent and adaptive space-time beampattern synthesis which provides both main beam and sidelobe control with reasonable computational complexity. Beampattern control is achieved by imposing a set of inequality constraints on the weighted mean-square error between the adaptive pattern and a desired beampattern over a set of angle-Doppler regions. An iterative procedure for satisfying the constraints is developed which can be applied as post-processing to standard MMSE beamformers. The algorithm is used to synthesize a nearly uniform sidelobe level quiescent pattern for the UESA, and to control sidelobe levels for the same array in an adaptive manner. Performance results using data provided by Lincoln Lab show that under low sample support conditions, sidelobes can be effectively suppressed while maintaining high signal-to-interference plus noise ratio, and deep nulls on clutter and interferers.

Contents

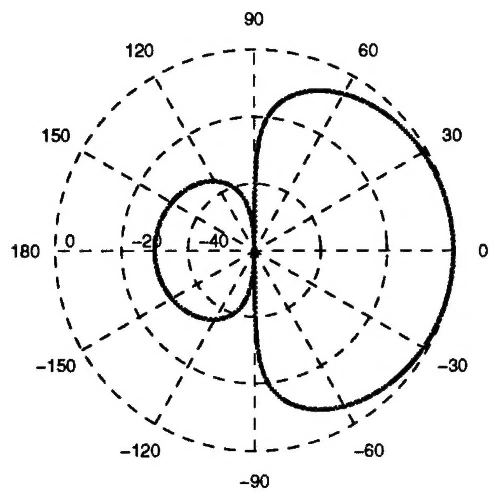
1 Introduction

Space-Time Adaptive Processing (STAP) used in airborne radar systems combines signals from N antenna array elements and M pulses to adaptively suppress clutter and jamming in both the space (angle) and time (Doppler frequency) dimension. The foundation of most STAP techniques is the Minimum Variance Distortionless Response (MVDR) processor [1]. The standard MVDR processor weights are designed to minimize the processor output power subject to a linear distortionless constraint in the angle-Doppler steering direction. The same weights can be obtained, to within a scale factor, using the minimum mean square error (MMSE) criterion, where the weights are designed to minimize the mean square error (MSE) between the processor output and a reference signal [2].

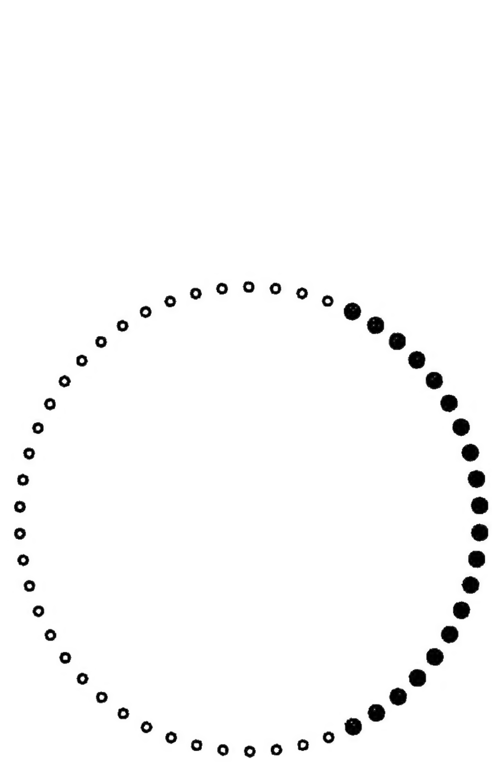
Traditionally, STAP systems have used a rotating linear array configuration, however a fixed circular ring array is currently under development under the UHF Electronically Scanned Array (UESA) program sponsored by the Office of Naval Research (ONR). The array consists of 54 directional antenna elements with suppressed backlobes [3]. The elements can be modeled as having a cosine pattern as shown in Figure 1(a). Only a portion of the elements will be used for each steering direction. A possible configuration is to use 20 of the elements at a time to transmit and receive [3], [4]. When the antenna has 54 elements, it can be scanned mechanically in $6.67°$ increments by choosing the appropriate 20-element sector, and scanned electronically $\pm 3.33°$ with the chosen sector of elements. This array configuration is illustrated in Figure 1(b). The resulting array spatial beampattern is shown in Figure 2(a) and the space-time beampattern is shown in Figure 2(b).

The circular array configuration has the potential to provide continuous $360°$ availability, however it has some potentially negative impacts for STAP algorithms [4]. First, the clutter rank is increased in a manner similar to the increase from misalignment with the velocity vector in linear arrays. Second, the clutter locus varies with range. This decreases the number of range gates that can be averaged to reliably estimate the clutter covariance matrix. In radar systems, this results in large sidelobes in the MVDR/MMSE beamformer which can lead to increased false alarms from clutter and unexpected interferers. Pattern control can be improved with steering vector tapering using a quiescent pattern with desirable main beam and sidelobe characteristics [5], linear main beam constraints [2], white noise gain constraints [6], quadratic quiescent pattern constraints [7]-[11], and reduced rank subspace techniques [12]-[14].

In this project, we have expanded on several of these techniques and developed a general framework

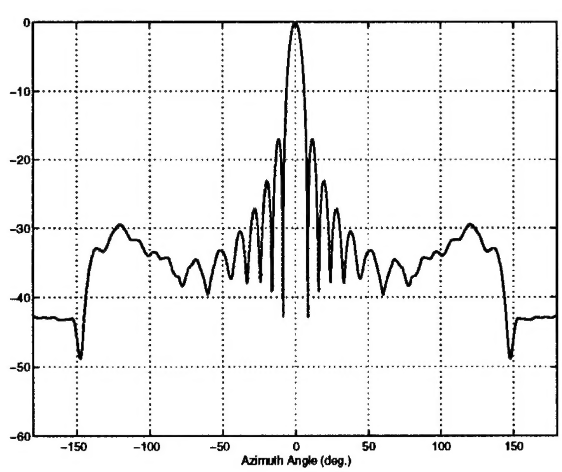


(a) Beampattern of endfire element modeled as cosine pattern with -30 dB backlobe

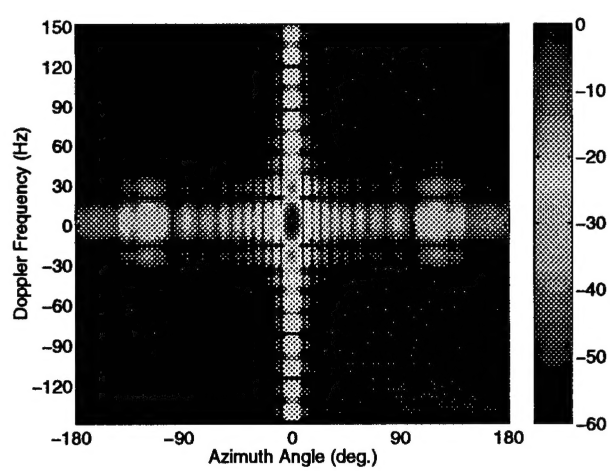


(b) Selection of 20 out of 54 active elements

Figure 1: UESA Circular Antenna Array



(a) Spatial Beampattern



(b) Space-Time Beampattern

Figure 2: UESA Conventional Beampatterns

for adaptive and non-adaptive beampattern synthesis for non-linear arrays based on minimum mean-square error (MMSE) beamforming with quadratic beampattern constraints (QPC). In this technique, main beam and sidelobe pattern control is achieved by imposing a set of inequality constraints on the weighted mean-square error between the adaptive pattern and a desired beampattern over a set of angle-Doppler regions. By proper choice of the number of constraints, the angular regions to which they apply, and the desired beampatterns in those regions, we can trade off the level of pattern control with algorithmic complexity. At one extreme, we can derive low-complexity beamforming techniques based on one or two constraints similar to the adaptive pattern control methods in [7]-[10]. At the other extreme, we can achieve tight pattern control using many constraints, in a manner similar to the technique in [11]. A more moderate approach using several constraints is shown to maintain a high SINR and good pattern behavior with reasonable complexity. The algorithm uses an iterative procedure for satisfying the constraints which can be applied as post-processing to standard STAP processors. The technique can also be used for non-adaptive pattern synthesis. It generalizes the techniques in [15] and [11] for developing low sidelobe quiescent patterns for arbitrary arrays, and can be used for developing the desired patterns used in the adaptive method. The technique has been applied to the UESA for both adaptive and non-adaptive pattern synthesis. Spatial beamforming results with the UESA circular array were published in [16] and CSTAP results will appear in [17] and [18].

This report is organized as follows. In Section 2, the MMSE-QPC problem formulation and optimum solution are developed, as well as the iterative implementation. In section 3, circular array STAP results using data provided by MIT Lincoln Lab [3] are presented. A summary and areas for future research are provided in Section 4.

2 MMSE Beamforming with Quadratic Pattern Constraints

2.1 Problem Formulation

We assume a STAP model with N antenna elements and M pulses. Let $\mathbf{v}(\theta, \phi, \omega)$ denote $NM \times 1$ space-time array response vector to a signal arriving with elevation angle θ, azimuth angle ϕ, and normalized Doppler frequency ω. We partition azimuth angle-Doppler space into r sectors, $\Omega_1, \ldots, \Omega_r$, as shown in Figure 3. In this illustration, the elevation angle space has only one partition and the sectors are cubes, however more general partitions of azimuth angle, elevation angle, and Doppler space can be used. Let $B_{d,i}(\theta, \phi, \omega) = \mathbf{w}_{d,i}^H \mathbf{v}(\theta, \phi, \omega)$ be a desired beampattern in the region Ω_i, and $\mathbf{w}_{d,i}$ be the corresponding

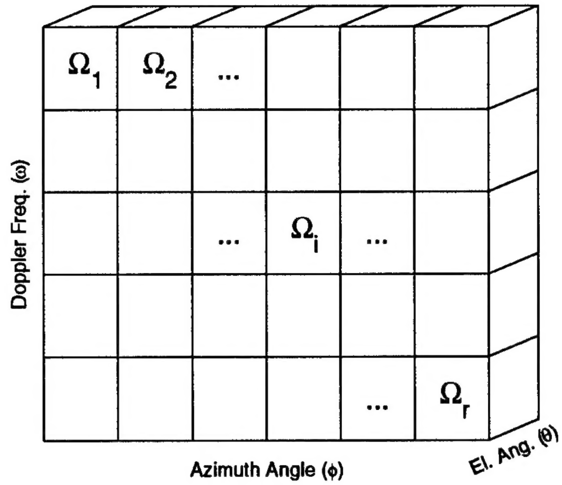


Figure 3: Partition of Angle-Doppler space

weight vector. The MSE between the beampattern generated by the adaptive weight vector $\mathbf{w}$ and the desired beampattern over the region Ω_i is given by

$$\epsilon_i^2 = \int_{\Omega_i} \left| \mathbf{w}^H \mathbf{v}(\theta, \phi, \omega) - \mathbf{w}_{d,i}^H \mathbf{v}(\theta, \phi, \omega) \right|^2 d\Omega. \tag{1}$$

The error can be written compactly as

$$\epsilon_i^2 = (\mathbf{w} - \mathbf{w}_{d,i})^H \mathbf{Q}_i (\mathbf{w} - \mathbf{w}_{d,i}) \tag{2}$$

where

$$\mathbf{Q}_i = \int_{\Omega_i} \mathbf{v}(\theta, \phi) \mathbf{v}(\theta, \phi)^H d\Omega. \tag{3}$$

Thus the pattern error is a quadratic function of the adaptive weight vector.

We design adaptive weights according to the standard MMSE beamforming criterion, while limiting the deviations from the desired pattern using quadratic pattern constraints. The MMSE-QPC optimization problem is

$$\begin{aligned} \min \quad & \mathbf{w}^H \mathbf{R}_{\mathbf{x}} \mathbf{w} - \mathbf{w}^H \mathbf{R}_{\mathbf{x}s} - \mathbf{R}_{\mathbf{x}s}^H \mathbf{w} + \sigma_s^2 \\ \text{st.} \quad & (\mathbf{w} - \mathbf{w}_{d,i})^H \mathbf{Q}_i (\mathbf{w} - \mathbf{w}_{d,i}) \leq L_i \quad i = 1, \ldots, r \end{aligned} \tag{4}$$

where $\mathbf{R}_{\mathbf{x}}$ is the data covariance matrix, $\mathbf{R}_{\mathbf{x}s}$ is the cross-correlation vector between the data and reference signal, and σ_s^2 is the reference signal power. When the reference signal is the desired signal, the cross-correlation vector has the form

$$\mathbf{R}_{\mathbf{x}s} = \sigma_s^2 \mathbf{v}_s. \tag{5}$$

The solution to the MMSE-QPC optimization problem has the form

$$\mathbf{w} = \left(\mathbf{R}_{\mathbf{x}} + \sum_{i=1}^{r} \lambda_i \mathbf{Q}_i\right)^{-1} \left(\sigma_s^2 \mathbf{v}_s + \sum_{i=1}^{r} \lambda_i \mathbf{Q}_i \mathbf{w}_{d,i}\right). \tag{6}$$

In the resulting beamformer, a weighted sum of 'loading' matrices $\mathbf{Q}_i, i = 1, \ldots, r$ are added to the sample covariance matrix, and a weighted sum of desired weight vector terms $\mathbf{Q}_i \mathbf{w}_{d,i}, i = 1, \ldots, r$ is added to the steering vector $\mathbf{v}_s$. The loading terms balance the adaptive pattern with the desired pattern. There are a set of optimum loading levels $\lambda_i, i = 1, \ldots, r$ which satisfy the constraints, however there is no closed form solution for the loading levels, even when $r = 1$. It can be shown that the mean-square pattern error decreases with increasing λ_i, but at the expense of decreased interference suppression. The loading levels must be chosen judiciously to achieve pattern control while maintaining high signal-to-interference-plus-noise ratio (SINR).

When the array is a uniformly spaced linear array, and there is one sector which covers the entire angular space, and the technique is the same as diagonal loading. When one or two quadratic constraints are imposed over the main beam and/or sidelobe regions, beamformers similar to those developed in [7]-[10] are obtained. These are relatively low complexity techniques that have been shown to be effective for pattern control when the loading levels are set appropriately. In practice, the optimum loading level is scenario dependent and it may be difficult to set properly to achieve the required pattern control.

When there is a single main beam constraint and many constraints over a dense grid of discrete points in the sidelobe region, the technique in [11] is obtained. This is a high complexity technique that achieves tight pattern control. By constraining the beampattern at a set of fixed points, logical constraint levels can be easily specified. If the desired pattern in the sidelobe region is set to zero, the pattern error is just the beampattern value, and the constraint level can be set to the to the desired sidelobe level. Constraints may also be imposed in the mainbeam region to maintain the mainbeam shape, although setting the constraint level isn't so straightforward.

We take a moderate approach using several constraints to obtain good pattern behavior with reasonable complexity. If the sectors are chosen small enough, we can still specify the desired pattern to be zero in the sidelobe region, and enforce maximum sidelobe levels by setting the constraint value to the desired sidelobe level times the volume of the sector. As the size of the sectors becomes larger, the pattern control is less exact, however the complexity is reduced. We have found that we can define a reasonable number of sectors that allow us to specify the constraints to maintain the sidelobe control we need, at a significantly reduced

complexity compared to the technique in [11].

2.2 Iterative Weight Computation

In [11], an iterative procedure is used to adjust the loading levels to achieve sidelobe level constraints. A similar procedure can be used to obtain the appropriate loading levels here. The algorithm can be initialized with the standard MMSE weights

$$\mathbf{w}^{(0)} = \sigma_s^2 \mathbf{R}_{\mathbf{x}}^{-1} \mathbf{v}_s, \tag{7}$$

or with tapered adaptive weights obtained by replacing $\sigma_s^2 \mathbf{v}_s$ with a tapered steering vector $\mathbf{w}_t$,

$$\mathbf{w}^{(0)} = \mathbf{R}_{\mathbf{x}}^{-1} \mathbf{w}_t. \tag{8}$$

At each iteration, let the weight error in each sector be

$$\mathbf{w}_{\epsilon,i}^{(p)} = \mathbf{w}^{(p-1)} - \mathbf{w}_{d,i}. \tag{9}$$

The pattern errors are computed and checked against the constraints. If a constraint is exceeded, i.e. if ${\mathbf{w}_{\epsilon,i}^{(p)}}^H \mathbf{Q}_i \mathbf{w}_{\epsilon,i}^{(p)} > L_i$, the loading for that sector, $\lambda_i^{(p)}$, is increased by $\Delta_i^{(p)}$ to $\lambda_i^{(p)} = \lambda_i^{(p-1)} + \Delta_i^{(p)}$, otherwise it remains at its current level. The covariance matrix and weights are updated by

$$\mathbf{R}_{\mathbf{x}}^{(p)} = \mathbf{R}_{\mathbf{x}} + \sum_{i=1}^{r} \lambda_i^{(p)} \mathbf{Q}_i \tag{10}$$

$$\mathbf{w}_t^{(p)} = \mathbf{w}_t + \sum_{i=1}^{r} \lambda_i^{(p)} \mathbf{Q}_i \mathbf{w}_{d,i} \tag{11}$$

$$\mathbf{w}^{(p)} = \left(\mathbf{R}_{\mathbf{x}}^{(p)}\right)^{-1} \mathbf{w}_t^{(p)}. \tag{12}$$

This is a computationally expensive procedure because the covariance matrix is inverted at each iteration. However, if the incremental loading levels $\Delta_i^{(p)}$ are small, the update can be accomplished without re-inverting the matrix by using a first order Taylor series approximation to the weight vector $\mathbf{w}^{(p)}$. Let $\mathbf{S}^{(p)}$ denote the covariance matrix inverse at each iteration,

$$\begin{aligned} \mathbf{S}^{(p)} &= \left(\mathbf{R}_{\mathbf{x}}^{(p)}\right)^{-1} = \left(\mathbf{R}_{\mathbf{x}}^{(p-1)} + \sum_{i=1}^{r} \Delta_i^{(p)} \mathbf{Q}_i\right)^{-1} \\ &= \left(\left(\mathbf{S}^{(p-1)}\right)^{-1} + \sum_{i=1}^{r} \Delta_i^{(p)} \mathbf{Q}_i\right)^{-1}. \end{aligned} \tag{13}$$

The steering vector at each iteration can be written as

$$\mathbf{w}_t^{(p)} = \mathbf{w}_t^{(p-1)} + \sum_{i=1}^{r} \Delta_i^{(p)} \mathbf{Q}_i \mathbf{w}_{d,i}. \tag{14}$$

Using (13) and (14) in (12), the adaptive weight vector is given by

$$\begin{aligned}\mathbf{w}^{(p)} &= \mathbf{S}^{(p)}\mathbf{w}_t^{(p)} \\ &= \left(\left(\mathbf{S}^{(p-1)}\right)^{-1} + \sum_{i=1}^{r}\Delta_i^{(p)}\mathbf{Q}_i\right)^{-1}\left(\mathbf{w}_t^{(p-1)} + \sum_{i=1}^{r}\Delta_i^{(p)}\mathbf{Q}_i\mathbf{w}_{d,i}\right). \qquad (15)\end{aligned}$$

The first order Taylor series approximation to $\mathbf{w}^{(p)}$ has the form

$$\mathbf{w}^{(p)} \approx \mathbf{w}_0^{(p)} + \mathbf{D}_0^{(p)}\mathbf{\Delta}^{(p)} \qquad (16)$$

where $\mathbf{\Delta}^{(p)}$ is the $r \times 1$ vector of loading increments, $\mathbf{D}_0^{(p)}$ is the derivative matrix evaluated at $\mathbf{\Delta}^{(p)} = \mathbf{0}$,

$$\mathbf{D}_0^{(p)} = \left[\frac{\partial \mathbf{w}^{(p)}}{\partial \Delta_1^{(p)}} \cdots \frac{\partial \mathbf{w}^{(p)}}{\partial \Delta_r^{(p)}}\right]\Bigg|_{\mathbf{\Delta}^{(p)}=\mathbf{0}} \qquad (17)$$

and $\mathbf{w}_0^{(p)}$ is the weight vector evaluated at $\mathbf{\Delta}^{(p)} = \mathbf{0}$,

$$\mathbf{w}_0^{(p)} = \mathbf{w}^{(p)}\Big|_{\mathbf{\Delta}^{(p)}=\mathbf{0}} = \mathbf{w}^{(p-1)}. \qquad (18)$$

The ith column of the derivative matrix is

$$\begin{aligned}\frac{\partial \mathbf{w}^{(p)}}{\partial \Delta_i^{(p)}} &= -\left(\left(\mathbf{S}^{(p-1)}\right)^{-1} + \sum_{i=1}^{r}\Delta_i^{(p)}\mathbf{Q}_i\right)^{-1}\mathbf{Q}_i\left(\left(\mathbf{S}^{(p-1)}\right)^{-1} + \sum_{i=1}^{r}\Delta_i^{(p)}\mathbf{Q}_i\right)^{-1} \bullet \\ &\quad \left(\mathbf{w}_t^{(p-1)} + \sum_{i=1}^{r}\Delta_i^{(p)}\mathbf{Q}_i\mathbf{w}_{d,i}\right) \\ &+ \left(\left(\mathbf{S}^{(p-1)}\right)^{-1} + \sum_{i=1}^{r}\Delta_i^{(p)}\mathbf{Q}_i\right)^{-1}\mathbf{Q}_i\mathbf{w}_{d,i} \qquad (19)\end{aligned}$$

Therefore

$$\begin{aligned}\frac{\partial \mathbf{w}^{(p)}}{\partial \Delta_i^{(p)}}\Bigg|_{\mathbf{\Delta}^{(p)}=\mathbf{0}} &= -\mathbf{S}^{(p-1)}\mathbf{Q}_i\left(\mathbf{S}^{(p-1)}\mathbf{w}_t^{(p-1)} - \mathbf{w}_{d,i}\right) \\ &= -\mathbf{S}^{(p-1)}\mathbf{Q}_i\left(\mathbf{w}^{(p-1)} - \mathbf{w}_{d,i}\right). \qquad (20)\end{aligned}$$

Combining (18), (20), and (9) with (16) yields

$$\mathbf{w}^{(p)} = \mathbf{w}^{(p-1)} - \mathbf{S}^{(p-1)}\sum_{i=1}^{r}\Delta_i^{(p)}\mathbf{Q}_i\mathbf{w}_{\epsilon,i}^{(p)}. \qquad (21)$$

The matrix inverse is updated is

$$\mathbf{S}^{(p)} = \mathbf{S}^{(p-1)} - \mathbf{S}^{(p-1)}\sum_{i=1}^{r}\Delta_i^{(p)}\mathbf{Q}_i\mathbf{S}^{(p-1)}. \qquad (22)$$

A simple way to achieve fast convergence while ensuring that the small update assumption is valid is to let $\Delta_i^{(p)}$ be a fraction of the of the current loading value, i.e. $\Delta_i^{(p)} = \alpha\lambda_i^{(p)}$, where α in the range 0.3 to 0.5 seems to work well. This requires that the initial loading level be non-zero. One possibility is to initialize all of the loading levels to some small value λ_0.

In summary, the algorithm is initialized by

$$\lambda_i^{(0)} = \lambda_0, i = 1, \ldots, r \tag{23}$$

$$\mathbf{S}^{(0)} = \left(\mathbf{R_x} + \lambda_0 \sum_{i=1}^{r} \mathbf{Q}_i\right)^{-1} \tag{24}$$

$$\mathbf{w}^{(0)} = \mathbf{S}^{(0)} \left(\mathbf{w}_t + \lambda_0 \sum_{i=1}^{r} \mathbf{Q}_i \mathbf{w}_{d,i}\right), \tag{25}$$

and the weights are updated by

1. for $i = 1, \ldots, r$

$$\mathbf{w}_{\epsilon,i}^{(p)} = \mathbf{w}^{(p-1)} - \mathbf{w}_{d,i} \tag{26}$$

$$\text{if } \mathbf{w}_{\epsilon,i}^{(p)H} \mathbf{Q}_i \mathbf{w}_{\epsilon,i}^{(p)} > L_i \quad \text{then } \Delta_i^{(p)} = \alpha\lambda_i^{(p)} \quad \text{else } \Delta_i^{(p)} = 0 \tag{27}$$

$$\lambda_i^{(p)} = \lambda_i^{(p-1)} + \Delta_i^{(p)} \tag{28}$$

2. $$\mathbf{Q}^{(p)} = \sum_{i=1}^{r} \Delta_i^{(p)} \mathbf{Q}_i \tag{29}$$

3. $$\mathbf{q}^{(p)} = \sum_{i=1}^{r} \Delta_i^{(p)} \mathbf{Q}_i \mathbf{w}_{\epsilon,i}^{(p)} \tag{30}$$

4. $$\mathbf{w}^{(p)} = \mathbf{w}^{(p-1)} - \mathbf{S}^{(p-1)} \mathbf{q}^{(p)} \tag{31}$$

5. $$\mathbf{S}^{(p)} = \mathbf{S}^{(p-1)} - \mathbf{S}^{(p-1)} \mathbf{Q}^{(p)} \mathbf{S}^{(p-1)}. \tag{32}$$

2.3 Non-Adaptive Pattern Synthesis

The technique can be used for non-adaptive pattern synthesis by letting $\mathbf{R_x} = \mathbf{I}$. It generalizes the techniques in [11] and [15] for developing low sidelobe quiescent patterns for arbitrary arrays, and can be used for developing the tapered steering vector used in the adaptive method.

3 Results

In the MIT Lincoln Lab data set [3], there are $N = 20$ elements and $M = 18$ pulses with a 300 Hz pulse repetition frequency. First, the MMSE-QPC technique was used to synthesize a -35 dB uniform sidelobe level quiescent pattern steered to $\phi = 0°$ and $\omega = 0$ Hz for a range of 50 km, which corresponds to $\theta =$

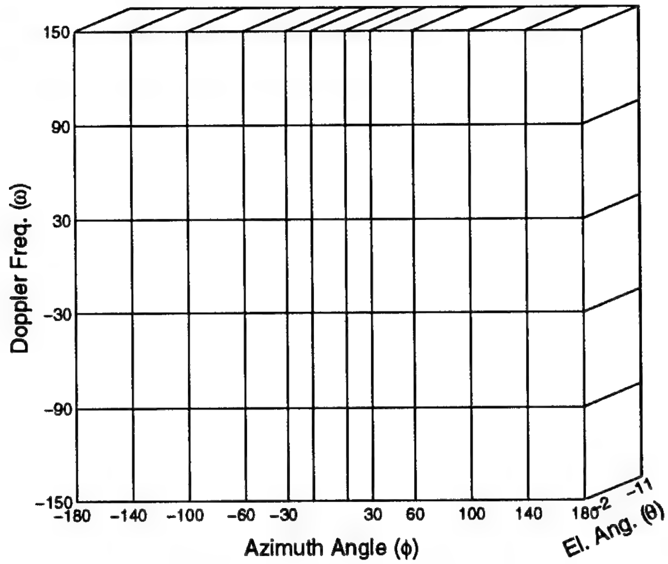


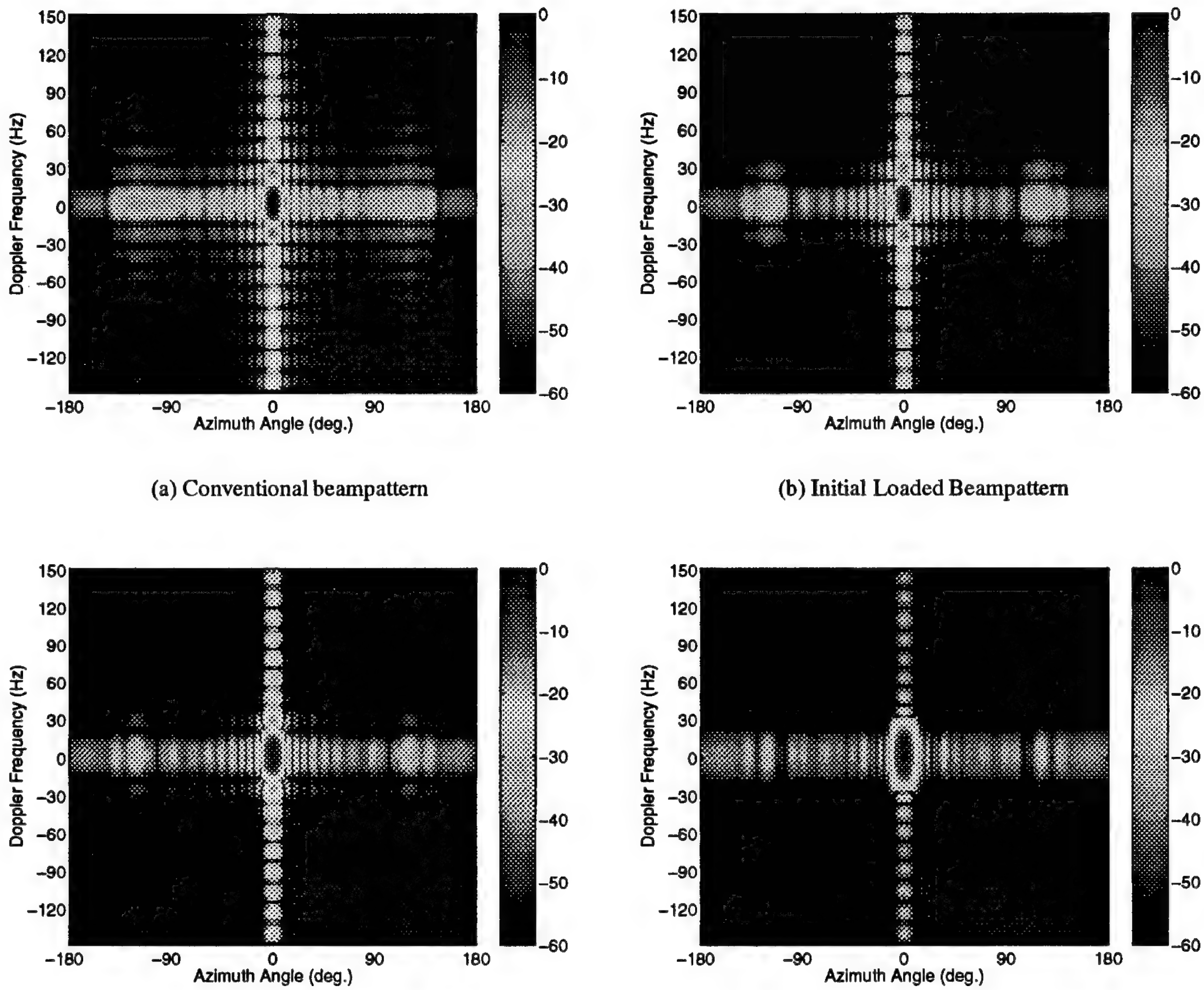
Figure 4: Partition of Angle-Doppler space for UESA

$-10.5°$. Angle-Doppler space was partitioned into one elevation angle sector $\theta \in (-11°, -2°)$, 11 azimuth angle sectors $\phi \in (-12°, 12°)$, $\pm(12°, 30°)$, $\pm(30°, 60°)$, $\pm(60°, 100°)$, $\pm(100°, 140°)$, $\pm(140°, 180°)$, and 5 Doppler sectors $\omega \in (-30, 30)$, $\pm(30, 90)$, $\pm(90, 150)$ Hz for a total of $1 \times 11 \times 5 = 55$ sectors, as shown in Figure 4.

The desired pattern was set to zero outside of the mainlobe region, and the constraint levels were set to -35 dB times the volume of the sector. No constraint was used in the mainlobe region. The loading levels were initially set to $\lambda_0 = 10$ dB, and then iteratively increased in the sectors where the constraint was not met using $\alpha = 1.5$. The exact weight update was used because this procedure can be done off-line. The algorithm converged in five iterations. The initial conventional space-time beampattern, and final -35 dB sidelobe level pattern are shown in Figure 5, along with several patterns at intermediate stages of the iteration. The final -35 dB sidelobe level pattern steered to $\phi = 0°, \omega = 60$ Hz is shown in Figure 6.

Next, a scenario with two 30 dB interference-to-noise ratio (INR) jammers at 60° and -20°, in addition to clutter, was considered. An 8 km training window (200 snapshots) was used to estimate the covariance matrix. The standard MMSE processor weights were computed by adding -30 dB diagonal loading to allow the covariance matrix to be inverted. The resulting space-time beampattern, and beampattern cuts are shown in Figures 7(a) and 7(b). The beamformer has put nulls on the clutter ridge and the two jammers, however the sidelobes are quite high.

The MMSE-QPC adaptive beamformer was used to reduce the sidelobes. The quiescent weights de-



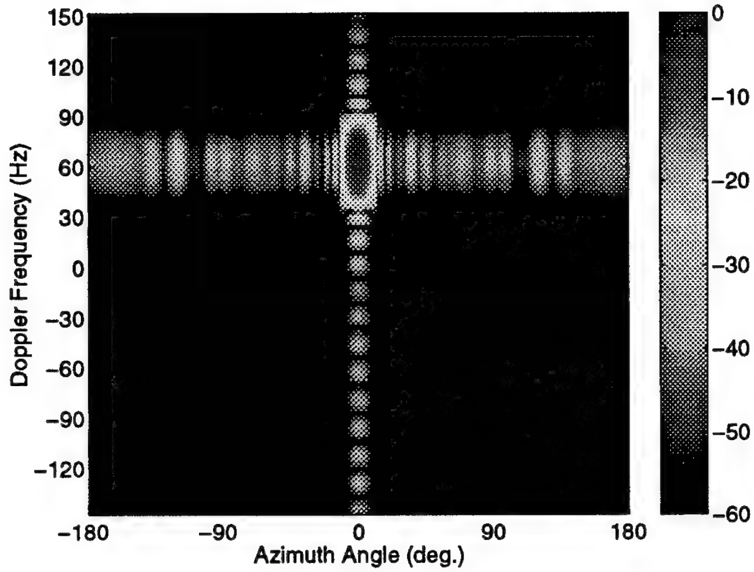
(a) Conventional beampattern

(b) Initial Loaded Beampattern

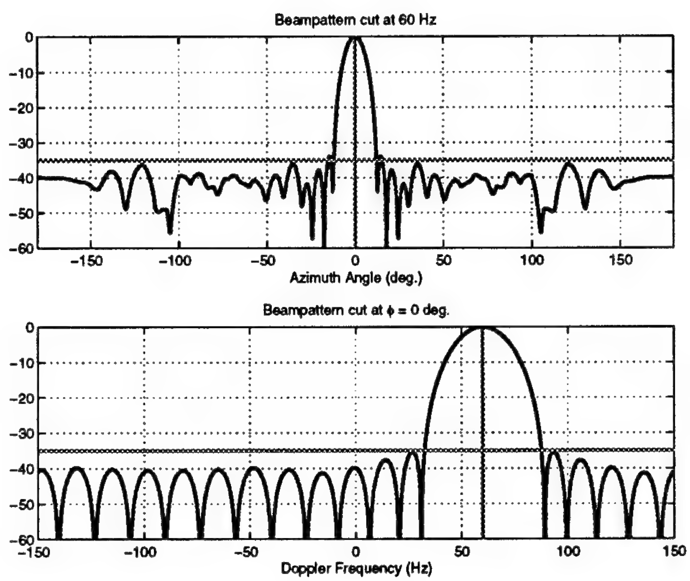
(c) Second Iteration

(d) Fifth Iteration

Figure 5: Quiescent Pattern Synthesis



(a) Space-Time Quiescent Beampattern



(b) Quiescent beampattern cuts

Figure 6: Low Sidelobe Level Quiescent Beampattern Steered to $\phi = 0°, \omega = 60Hz$

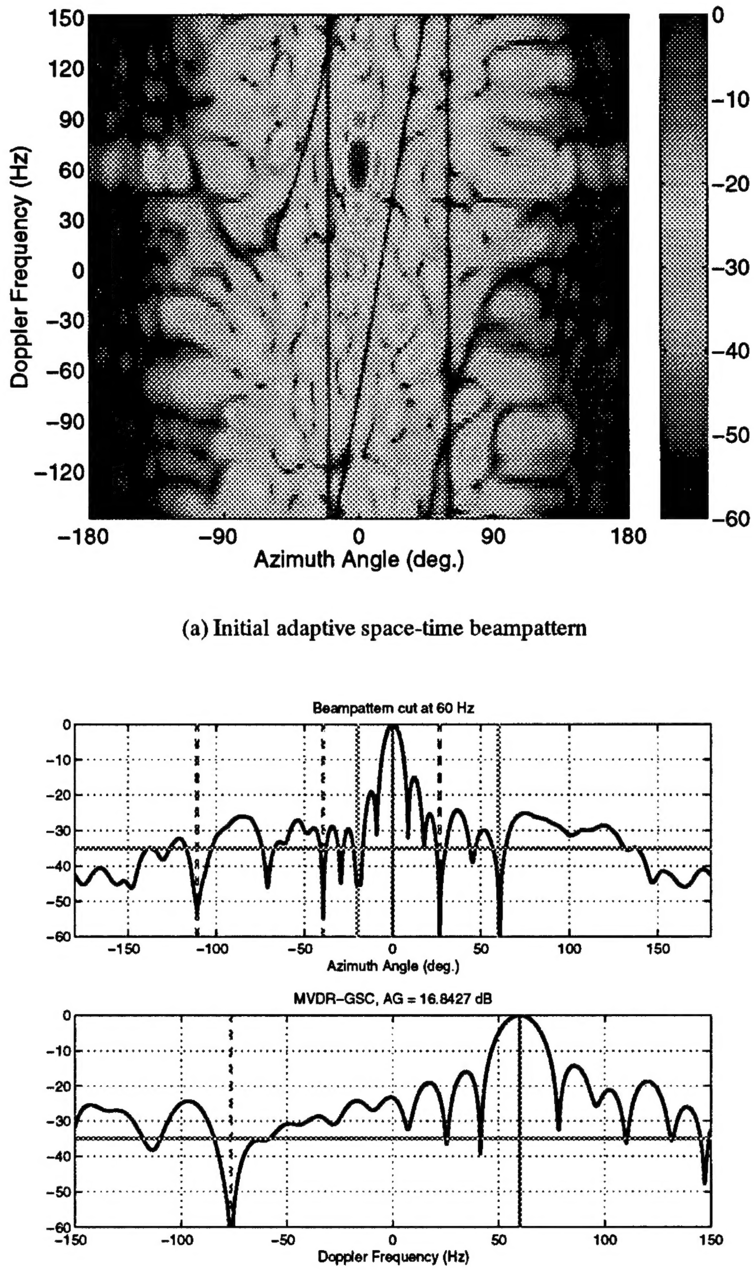
rived in the first example was used as the tapered steering vector. The initial loading levels were set to $\lambda_0 = 0.0013$, and then iteratively increased using $\alpha = 0.8$. In six iterations, the MMSE-QPC is able to reduce the sidelobes below the -35 dB level while maintaining a well behaved main-beam, and deep clutter and jammer nulls. The final beampattern is shown in Figures 8(a) and 8(b).

4 Summary and Future Research

The MMSE-QPC technique provides a general framework for quiescent and adaptive space-time beampattern synthesis which provides both main beam and sidelobe control with reasonable computational complexity. Beampattern control is achieved by imposing a set of inequality constraints on the weighted mean-square error between the adaptive pattern and a desired beampattern over a set of angle-Doppler regions. An iterative procedure for satisfying the constraints was developed which can be applied as post-processing to standard MMSE beamformers, and guidelines for setting constraint levels and algorithm parameters to achieve desired performance were discussed.

The algorithm was used to synthesize a nearly uniform sidelobe level quiescent space-time beampattern for the UESA, and to control sidelobe levels for the same array in an adaptive manner. Performance results using data provided by Lincoln Lab showed that under low sample support conditions, sidelobes could be effectively suppressed while maintaining high signal-to-interference plus noise ratio, and deep nulls on clutter and interferers.

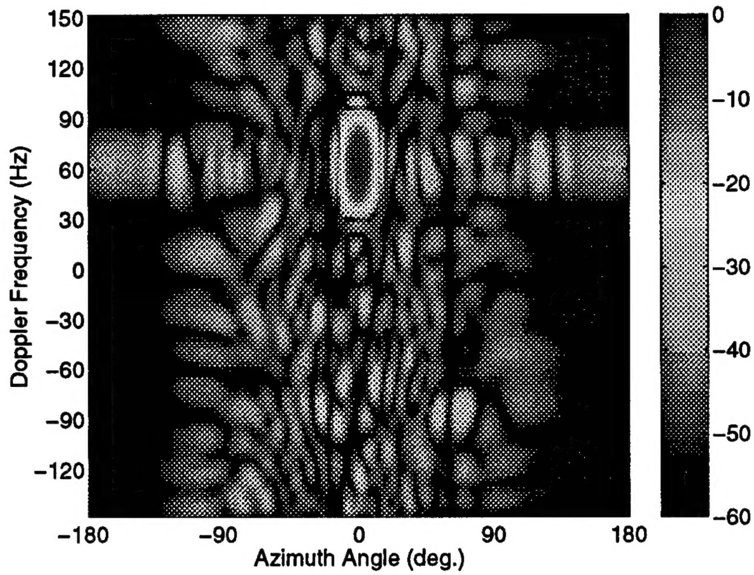
This formulation can be extended in a straightforward manner to the MVDR and more general linearly constrained minimum variance (LCMV) beamforming criteria. It can also be used in other fully and partially adaptive STAP architectures. Topics to be pursued under the second phase of the project include applying the quadratic pattern constraint methodology to LCMV beamforming; generalizing the technique for fully and partially adaptive STAP architectures in element and beam space, and with pre- and post-Doppler processing; applying the technique to the 3D STAP model for hot clutter mitigation; and evaluating subsequent adaptive detection performance.



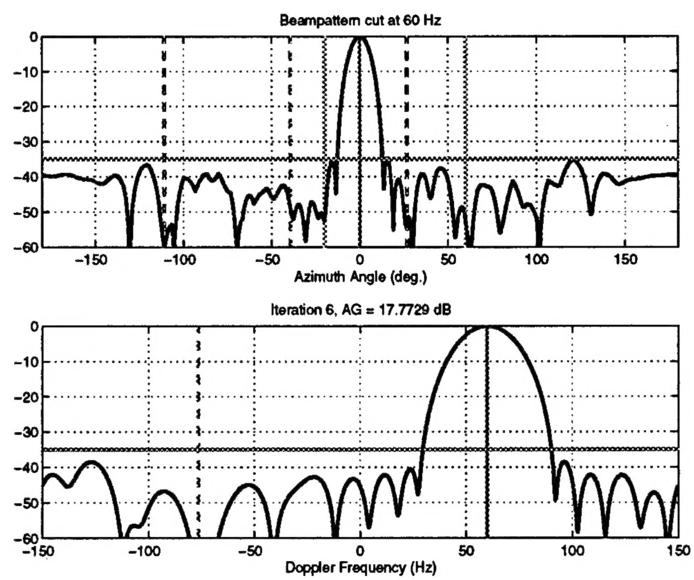
(a) Initial adaptive space-time beampattern

(b) Initial adaptive beampattern cuts

Figure 7: Initial Adaptive Beampattern Steered to $\phi = 0°, \omega = 60$ Hz.



(a) Adaptive space-time beampattern after sixth iteration



(b) Sixth Iteration beampattern cuts

Figure 8: Final Adaptive Beampattern Steered to $\phi = 0°, \omega = 60$ Hz.

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14. ABSTRACT
The goal of this project was to develop new and innovative processing methods for clutter and interference mitigation for Space-Time Adaptive Processing (STAP) with circular arrays. The focus was on developing a robust minimum mean square error (MMSE) beamforming technique using quadratic quiescent pattern constraints (MMSE-QPC) that works with arbitrary array configurations, including the circular UHF Electronically Scanned Array (UESA) and the standard linear array. The MMSE-QPC technique provides a general framework for quiescent and adaptive space-time beampattern synthesis which provides both main beam and sidelobe control with reasonable computational complexity. Beampattern control is achieved by imposing a set of inequality constraints on the weighted mean-square error between the adaptive pattern and a desired beampattern over a set of angle-Doppler regions. An iterative procedure for satisfying the constraints is developed which can be applied as post-processing to standard MMSE beamformers. The algorithm is used to synthesize a nearly uniform sidelobe level quiescent pattern for the UESA, and to control sidelobe levels for the same array in an adaptive manner. Performance results using data provided by Lincoln Lab show that under low sample support conditions, sidelobes can be effectively suppressed while maintaining high signal-to-interference plus noise ratio, and deep nulls on clutter and interferers.

15. SUBJECT TERMS
Space-Time Adaptive Processing, circular arrays, beamforming, quadratic constraints, sidelobes

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